Unclassified/Unlimited

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

AD-A221 735

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER: No. 38
2. GOVT ACCESSION NO.:
3. RECIPIENT'S CATALOG NUMBER:
4. TITLE (and Subtitle): Photon Statistical Properties of an Optical Wiggler Free Electron Laser
5. TYPE OF REPORT & PERIOD COVERED: Technical Report 10/01/84 - 09/30/85
6. PERFORMING ORG. REPORT NUMBER:

AUTHOR(s): G. Dattoli and J. Gallardo

8. CONTRACT OR GRANT NUMBER(s): N00014-80-C-0308

PERFORMING ORGANIZATION NAME AND ADDRESS: University of California, Quantum Institute, Santa Barbara, CA 93106

10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS: 61153N; RR011-07-06; NR603-001

CONTROLLING OFFICE NAME AND ADDRESS: ONR, 1030 E. Green St., Pasadena, CA 91106

12. REPORT DATE: 1985

NUMBER OF PAGES:

MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office):

15. SECURITY CLASS. (of this report): Unclassified/Unlimited

15a. DECLASSIFICATION/DOWNGRADING SCHEDULE:

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

Journal of the Optical Science B, 1985.

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Photon, Properties, Wiggler, FEL

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

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Photon statistical properties of an

optical wiggler Free Electron Laser

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ABSTRACT: We use the Free Electron Laser Hamiltonian to study the statistical properties of the photons emitted in a scattering process between a coherent electromagnetic wave (optical wiggler) and a counter-propagating relativistic electron beam. It is shown that the probability distribution of exchanged photons is binomial and that coherent states are preserved by the Free Electron Laser interaction only in the absence of gain.

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1. INTRODUCTION.

In a Free Electron Laser (FEL) device an electron beam interacts with a permanent magnet wiggler. The magnetic field can be treated as a real electromagnetic wave according to the prescription of Weizsäcker-Williams. In the quantized version of the FEL interaction it is assumed that the wiggler is a classical wave with a fixed but infinitely large number of photons. In this short note we will assume the wiggler to be a real electromagnetic field described by a coherent state. This problem was suggested by the fact that coherence in the Glauber sense is not preserved by the FEL interaction even at zero electron recoil if the wiggler is a classical wave and the input laser field is a coherent wave[1] ;furthermore, the photon distribution was shown to be sub-Poissonian. We now ask the equivalent questions: does the laser field remain in a coherent state during the interaction when the wiggler is in a coherent state and there are not laser photons present at $t = 0$?; what does the photon distribution look like?. These questions have relevance in fully understanding coherence in an FEL. Our treatment can be extended to understand other problems in Quantum Optics, i.e. the evolution of coupled harmonic oscillators and consequently that of coherent Bloch states[2].

2. THEORY.

The single mode, *single particle non-relativistic* Hamiltonian describing the FEL process reads

$$H = \frac{P^2}{2m} + \hbar\omega\left(a_L^\dagger a_L + a_u^\dagger a_u + 1\right) + \hbar\Omega\left[a_L^\dagger a_u e^{-2ikz} + a_u^\dagger a_L e^{2ikz}\right] \qquad (1)$$

where P, z are the electron linear momentum and longitudinal position, respectively; $a_L^\dagger(a_u^\dagger)$ is the laser (undulator) creation operators and $\Omega = \frac{2\pi e^2}{V m\omega}$ the coupling constant with V the interaction volume.

From Eq. (1) it can be readily seen that the total linear momentum and the number of photons are conserved quantities[1]. In the assumption of both laser and undulator initial fields are in coherent states with mean number of photons $|\alpha_o|^2, |\beta_o|^2$ respectively, the state of the system at time t is given by

$$|\Psi(t)\rangle = e^{-it\frac{P_o^2}{2m\hbar}-it\omega}\sum_{n=0}^{\infty}\frac{e^{-\frac{1}{2}|\alpha_o|^2}}{\sqrt{n!}}\alpha_o^n\sum_{m=0}^{\infty}\frac{e^{-\frac{1}{2}|\beta_o|^2}}{\sqrt{m!}}\beta_o^m$$

$$\times\sum_{l=-n}^{m} C_l^{m,n}(t)|P_o - 2lk\hbar, n+l, m-l\rangle \qquad (2)$$

where P_o is the initial electron momentum, l is an integer enumerating the exchanged photons and $C_l^{m,n}(t)$ is the probability amplitude for interchanging l-photons in the presence of m and n undulator and laser photons, respectively. Replacing Eq. (2) in the Schrödinger equation we obtain a differential-difference equation of the spherical Raman-Nath type[3]

$$i\frac{d}{d\tau}C_l^{m,n}(\tau) = (-W_o + \epsilon l)lC_l^{m,n}(\tau) + \Omega_R\left[\sqrt{(n+l+1)(m-l)}C_{l+1}^{m,n}(\tau)\right.$$

$$+\sqrt{(n+l)(m-l+1)}C^{m,n}_{l-1}(\tau)\Big] \tag{3}$$

with the initial condition $C_l^{m,n}(0) = \delta_{l,0}$, where τ is the dimensionless time ($\tau = \gamma\frac{ct}{L}$), $W_o = \omega 2 P_o L/mc^2$ is the resonance parameter, $\Omega_R = \Omega L/c$ is the coupling constant and $\epsilon = 2\hbar k^2 L/mc$ is the electron recoil; ϵ is of the order of 10^{-7} for most cases of interest.

Choosing a vacuum as input field ($|\alpha_o|^2 = 0$), the solution of the Raman-Nath Equation[3] in first order in ϵ reads

$$C_l^m(\tau) = e^{il\Delta(\tau) - im\Phi(\tau)}[A_l^m(\tau) + iD_l^m(\tau)] \tag{4}$$

where $\Delta(\tau)$ and $\Phi(\tau)$, which we need not to specify here, have been given in Ref. 3. The functions appearing in Eq. (4) are given by

$$A_l^m(\tau) = B_l^m(\tau) + \epsilon\Omega_R \frac{\partial}{\partial W_o}\left(\frac{\sin W_o\tau/2}{W_o/2}\right)\Big\{(2l+1)\sqrt{(l+1)(m-l)}B^m_{l+1}(\tau)$$

$$-(2l-1)\sqrt{l(m-l+1)}B^m_{l-1}(\tau)\Big\} - \frac{1}{2}\epsilon\Omega_R^2\frac{\partial}{\partial W_o}\left(\frac{\sin W_o\tau/2}{W_o/2}\right)^2$$

$$\times\Big\{\sqrt{(l+1)(l+2)(m-l)(m-l-1)}B^m_{l+2}(\tau) - \sqrt{l(l-1)(m-l+1)(m-l+2)}B^m_{l-2}(\tau)\Big\} + \dots \tag{5a}$$

and

$$D_l^m(\tau) = \frac{1}{2}\epsilon\Omega_R\tau\left(\frac{\sin W_o\tau/2}{W_o/2}\right)\Big\{(2l+1)\sqrt{(l+1)(m-l)}B^m_{l+1}(\tau)$$

$$+(2l-1)\sqrt{l(m-l+1)}B^m_{l-1}\Big\} - \frac{1}{2}\epsilon\Omega_R^2\tau\left(\frac{\sin W_o\tau/2}{W_o/2}\right)^2$$

$$\times\Big\{\sqrt{(l+1)(l+2)(m-l)(m-l-1)}B^m_{l+2}(\tau) + \sqrt{l(l-1)(m-l+1)(m-l+2)}B^m_{l-2}(\tau)\Big\} + \dots \tag{5b}$$

where we have denoted the binomial distribution as

$$B_l^m(\tau) = \sqrt{\binom{m}{l}}p^{l/2}(1-p)^{(m-l)/2} \tag{6a}$$

with

$$p = \frac{4\Omega_R^2}{(4\Omega_R^2 + W_o^2)}\sin\frac{\tau}{2}\sqrt{4\Omega_R^2 + W_o^2} \tag{6b}$$

An interesting question is the definition of coherent state in the context of this model and within the $\epsilon = 0$ approximation. *The most obvious difficulty we are* faced with is how to deal with the electron variables. Now, since the Hamiltonian contains two coupled (through e^{2ikz}) harmonic oscillators, we can follow the definition suggested in Ref. 4 for the coherent Bloch states, namely a simultaneous eigenstate of the laser and undulator destruction operator. This suggests to define the *BOSON* operators

$$A = e^{2ikz}\frac{a_L a_s}{\sqrt{|\alpha_o|^2 + |\beta_o|^2}} \tag{7a}$$

with the commutation relation

$$\langle\Psi|\left[A, A^\dagger\right]|\Psi\rangle = 1 \tag{7b}$$

It is not difficult to prove that $|\Psi(\tau)\rangle$ is an eigenstate of A and the average value is given by

$$\langle\Psi(\tau)|A|\Psi(\tau)\rangle_{\epsilon=0} = \sqrt{p(1-p)}|\beta_o|^2$$

Hence, the state $|\Psi\rangle$ of Eq. (2) for zero electron recoil is a coherent Bloch state in the sense of Ref. 4. Clearly, for $\epsilon \neq 0$ the FEL interaction does not preserve coherence. Next, we evaluate the average value of the exchanged photons

$$\langle l\rangle_{\epsilon\neq0} = |\beta_o|^2\left\{p - 2\epsilon\Omega_R\sqrt{p(1-p)}\frac{\partial}{\partial W_o}\left(\frac{\sin W_o\tau/2}{W_o/2}\right)\right\} + \dots \tag{8}$$

where p *is given in Eq.* (6b)*; the additional terms are higher order in* Ω_R . This result is obtained using recursive properties of the binomial distribution. In the limit of $|\beta_o|^2 \to \infty$ and $\Omega_R \to 0$, with $|\beta_o|^2\Omega_R \equiv \tilde{\Omega} = constant$, Eq. (8) becomes

$$\langle l\rangle_{\epsilon\neq0} \longrightarrow \tilde{\Omega}^2\left(\frac{\sin W_o\tau/2}{W_o/2}\right)^2 - \epsilon\tilde{\Omega}^2\frac{\partial}{\partial W_o}\left(\frac{\sin W_o\tau/2}{W_o/2}\right)^2 + \dots$$

which is the usual expression for the FEL gain[1] with permanent magnet undulator. We also evaluate

$$\langle\Delta l^2\rangle_{\epsilon\neq0} - \langle l\rangle_{\epsilon\neq0} \approx O(\Omega_R^4)$$

quantity which measures the departure from Poisson statistics[6], finding agreement with previous results[1,5].

In conclusion, we have shown that in a parametric process involving the FEL interaction of a single electron with a coherent wave (optical wiggler) the spontaneously emitted photons distribution function is binomial (Eq. (6)). We have also shown that in first order in the electron recoil the average number of exchanged photon contains, besides the spontaneous emission term, a stimulated emission factor similar to the one appearing in the undulator FEL process.

ACKNOWLEDGEMENT

The authors acknowledge discussion with L. Elias. They also thank P. Burris for her assistance in the preparation of the manuscript. (G.D.) is grateful to the FEL group of the Quantum Institute, University of California at Santa Barbara for partial financial support. (J.G.) has been supported by the Office of Naval Research, ONR Contract NOOO14-80-C-0308.

REFERENCES

1. G. Dattoli, J. Gallardo, A. Renieri and M. Richetta, to appear,*IEEE, J. of Quantum Elect.*, special issue on Free Electron Lasers QE/July(1985).

2. F. T. Arecchi, E. Courtens, G. Gilmore and H. Thomas, *Phys. Rev.* *A* **6**, 2211, (1972); R. Bonifacio, D. M. Kim and M. Scully, *Phys. Rev.* **187**, 441, (1969).

3. P. Bosco, G. Dattoli, and J. Gallardo, *J. Phys.* *A* **17**, 2739, (1984).

4. G. Dattoli, A. Renieri, F. Romanelli and R. Bonifacio, *Opt. Comm.* **34**, 240, (1980); see also Ref. 2

5. W. Becker and J. K. McIver, *Phys. Rev.* *A* **27**, 1030, (1983); *Many Particle Quantum Theory for a Class of Free Electron Devices* to be published in **Topics on Current Physics**, Springer-Verlag, New York (1985).

6. L. Mandel, *Opt. Lett.* **4**, 205, (1979).